

# Chirality induced anomalous-Hall effect in helical spin crystals

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## Abstract

Under pressure, the itinerant helimagnet MnSi displays unusual magnetic properties. We have previously discussed a BCC helical spin crystal as a promising starting point for describing the high pressure phenomenology. This state has topologically nontrivial configurations of the magnetization field. Here we note the consequences for magneto-transport that arise generally from such spin textures. In particular a skyrmion density induced ‘topological’ Hall effect, with unusual field dependence, is described.

*Key words:* , helimagnets; magnetotransport; skyrmions; anomalous Hall effect

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## 1. Introduction

Recently, the interplay between electron transport and magnetism has been at the center of renewed activity. The anomalous hall effect in ferromagnets is a prototypical example of this interplay. Furthermore, transport in the presence of more complicated magnetic order, such as spin spirals, are proving to be extremely rich. An interesting analog of the anomalous Hall effect of ferromagnets, the ‘topological’ Hall effect, is expected [1] where Berry’s phase from skyrmion spin textures leads to an effective magnetic flux. Experimentally, novel physical phenomena have also been reported in itinerant helimagnets (with spiral magnetic order), especially in the B20 structure family which includes FeGe and MnSi. The latter was long regarded as an example of a well understood magnetic system, until recent experiments under moderate pressure revealed a quantum phase transition into a ‘partial order’ state, with an unusual signature in neutron scattering and non-Fermi liquid resistivity signature  $\Delta\rho(T) \propto T^{1.5}$  [2,3]. This non-Fermi liquid resistivity is seen for almost three decades in temperature from around 6K down to a few mK. While the unconventional magnetism disappears on further increasing the pressure, the non-fermi liquid signatures in transport persists up to the highest pressures studied. Similar phenomena are seen

in the related compound FeGe, where spin spirals are also believed to play a role [4].

Recently, the anomalous Hall effect has been measured by several groups both at ambient and higher pressures [5]. M. Lee et al. have identified a sharp feature in the Hall resistivity, which appears to be increased in a restricted parameter region of temperature, pressure and field [6]. They have put forward the interesting speculation that this feature may be related to a ‘topological’ Hall effect (or Hall effect produced by spin chirality) [1].

Here, we first review our proposal of BCC helical spin crystals as a starting point to describing the ‘partial order’ state [7,8]. These and other spin crystals may be understood as periodic arrangements of skyrmion configurations. The physical manifestations of such topologically non-trivial spin textures is the main subject of this article. In particular, we discuss the possibility of observing a topological Hall effect in such textures, form two simple examples.

*Phenomenological theory of anomalous magnetism in MnSi at high pressure:* MnSi is found to have the following phase diagram. At ambient pressure, a ferromagnetic ordering occurs below 30K. The presence of weak Dzyaloshinskii-Moriya interactions which are allowed by the non-centrosymmetric B20 cubic structure, leads to a spiraling of the magnetization. A single spiral state with a pitch of about  $\lambda \approx 170\text{\AA}$  and a small moment  $0.3\mu_B$  is produced. Crystalline anisotropy locks the spiral along certain fixed crystal directions - a simple symmetry anal-

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ysis reveals that the simplest energy terms consistent with the B20 crystalline symmetries are such that they favor the spiral along the  $\langle 111 \rangle$  direction or the  $\langle 100 \rangle$  directions. Indeed, neutron scattering at ambient pressure reveals Bragg spots along the  $\langle 111 \rangle$  direction (where  $\langle 111 \rangle$  refers to the  $[111]$  and the seven other crystallographically equivalent directions). On increasing the pressure beyond  $p_c = 14.6 \text{ kBar}$ , there is a quantum phase transition into an unusual state termed the 'partial order' state in Ref. [3]. Here, in contrast to ambient pressure, the elastic neutron scattering reveals enhanced scattering all around the wavevector sphere  $\mathbf{k} = 2\pi/\lambda$ . However the intensities are peaked around the  $\langle 110 \rangle$  directions. It may be argued that this is very unlikely to be due to single spiral states [7,8].

We therefore adopt the next simplest hypothesis, that a useful starting point to explain partial order is a spin crystal, made of a coherent superposition of magnetic spirals along the  $\langle 110 \rangle$  directions. Approaches which are complementary to ours Refs. [9,10], have proposed different periodic spin textures starting from real space rather momentum space. At longer scales, magnetic order is believed to be destroyed by disorder or fluctuation effects. Experimentally, while NMR sees a frozen magnetic moment in this regime [11],  $\mu\text{SR}$  does not [12]. Here, we assume the order is static, at least in the presence of a magnetic field.

Elsewhere, we have studied the energetics of such a state relative to the single mode state [7,8]. Spatially varying interactions between spirals are required to stabilize such a state, and is more likely the shorter the spiral wavelength. In this respect, although MnSi has a relatively long spiral wavevector, it is the shortest compared to others in the same family (e.g. FeGe, FeCoSi etc.). Here we do not revisit the energetics, but rather adopt a phenomenological approach and deduce the consequences of having a  $\langle 110 \rangle$  spin crystal. Note, the most general such spin crystal can be specified by the amplitude and phase of the spirals along each direction. This requires specifying six complex numbers  $\psi_1, \psi_2 \dots \psi_6$ . From these, and a specification of the basis used along each spiral direction, one can reconstruct  $\mathbf{M}(\mathbf{r})$ . If we choose the six wavevectors  $\mathbf{k}_j$  to be along  $\{(1, \bar{1}, 0), (\bar{1}, 1, 0), (0, \bar{1}, 1), (0, 1, \bar{1}), (1, 0, 1), (1, 0, \bar{1})\}$ , and the basis vectors to be unit vectors  $\hat{\epsilon}_j'' \propto [(0, 0, 1) \times \mathbf{k}_j]$ , and  $\hat{\epsilon}_j' \propto [\hat{\epsilon}_j'' \times \mathbf{k}_j]$ , we can write:

$$\mathbf{M}(\mathbf{r}) = \frac{1}{2} \sum_{j=1}^6 \psi_j (\epsilon_j' + i\epsilon_j'') e^{i\mathbf{k} \cdot \mathbf{r}} + h.c. \quad (1)$$

These spin structures have the periodicity of a body centered cubic (bcc) lattice, hence, bcc spin crystals.

We can now develop a Landau theory in terms of these complex fields. Cubic symmetry restricts the Landau free energy to the following form:

$$F = \sum_{jj'} V_{jj'} |\psi_j|^2 |\psi_{j'}|^2 + \lambda \text{Re}(T_x + T_y + T_z), \quad (2)$$

where  $T_x = \psi_1^* \psi_2 \psi_5 \psi_6$ ,  $T_y = \psi_1^* \psi_2^* \psi_3 \psi_4$  and  $T_z = -\psi_3 \psi_4^* \psi_5^* \psi_6$ . The bcc state is realized if the parameters  $V_{ij}$

and  $\lambda$  are such that the energy minimum has equal amplitudes for all six modes. As for the phases, it is clear, that they only depend on the sign of the parameter  $\lambda$ . For  $\lambda > 0$  ( $< 0$ ), the phases arrange in such a way that  $T_x, T_y, T_z$  are negative (positive). The remaining three phase degrees of freedom correspond to global translations of the structure. Thus symmetry considerations alone lead to just two possible bcc states, called bcc1 and bcc2 respectively. Both states are doubly degenerate due to time reversal  $\mathbf{M} \rightarrow -\mathbf{M}$ . The state bcc1 ( $\lambda > 0$ ) is more likely to be realized than bcc2, because the magnitude of magnetic moments  $|\mathbf{M}(\mathbf{r})|$  has a much narrower distribution in bcc1 than in bcc2 [7]. The resulting real-space pattern  $\mathbf{M}(\mathbf{r})$  of bcc1 is best understood as a periodic network of skyrmion lines and anti-vortex node-lines [7,8]. These structures are stabilized by symmetry - the magnetic point group symmetry of the structure is  $O(T)$ . This group is generated by  $\pi/2$  rotations around the  $x$ -,  $y$ - or  $z$ -axis, followed by time-reversal [7,8]. We will see that the most direct manifestation of these structures is in the magneto-transport, leading to a 'topological' Hall effect.

## 2. Skyrmion density

Non-trivial magnetic structures in metals can act as an effective orbital magnetic field on conduction electrons and thus lead to an anomalous Hall effect which is independent of spin-orbit coupling [1]. Essentially, when conduction electrons are forced to follow the local spin direction  $\mathbf{M}(\mathbf{r})$ , they acquire a Berry's phase. This may be viewed as an effective field, which is proportional to the skyrmion-density  $\Phi(\mathbf{r})$ , defined as

$$\Phi^\alpha = \frac{1}{8\pi} \epsilon^{\alpha\beta\gamma} \hat{\mathbf{n}} \cdot (\partial_\beta \hat{\mathbf{n}} \times \partial_\gamma \hat{\mathbf{n}}), \quad (3)$$

where  $\hat{\mathbf{n}} = \mathbf{M}/|\mathbf{M}|$ . The flux of  $\Phi$  through any two-dimensional rectangle with periodic boundary conditions is a topological winding number, i.e. an integer. Note, the terminology 'spin chirality' is somewhat misleading since  $\Phi$  transforms as an axial vector, like the magnetic field. It is *even* under inversion, hence not a chiral object - i.e. broken inversion symmetry is not required to have  $\Phi \neq 0$ .

It is remarkable that so many different theoretical proposals for novel magnetic structures in 3D magnets without inversion symmetry independently lead to arrangements of skyrmion lines (meron lines, double-twist cylinders) [7,8,9,10]. Therefore, such systems appear to be natural candidates for observing the effects of  $\Phi$ .

By inspection of Eq. (3), it becomes immediately clear that a simple spin density wave, which involves a single pair of wavevectors  $\pm \mathbf{k}$ , always has a *vanishing* skyrmion density, since in this case  $\mathbf{M}$  depends only on a single space coordinate. This is also true if  $\mathbf{M}$  acquires a uniform component, i.e. in the conical phase under a magnetic field. Multi-q spin crystals are therefore the simplest slowly varying magnetic structures with non-vanishing skyrmion density.

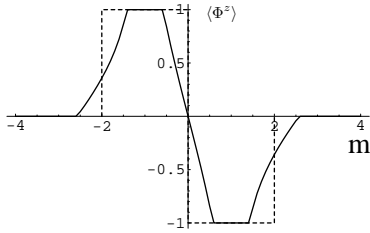


Fig. 1. The dotted line shows the skyrmion number  $\int dxdy\Phi^z$  of one unit cell for the helical 2D square spin crystal with a uniform component  $\mathbf{M}(\mathbf{r}) = (\sin y, \sin x, m + \cos y - \cos x)$  as a function of  $m$ . The full line shows the same quantity evaluated with a cutoff for  $\mathbf{M}^2 < 1/3$ , as explained in the text.

### 2.1. Square spin crystal

As the simplest example for helical spin crystal, we first consider the magnetic texture obtained by superimposing two orthogonal spirals of the same chirality. For simplicity, we choose units such that each spiral has unit amplitude and a wave-length equal to  $2\pi$ . In appropriate coordinates, the square spin crystal is then given by

$$\mathbf{M}(\mathbf{r}) = (\sin y, \sin x, \cos y - \cos x) \quad (4)$$

Note, it is a 2D like structure given that there is no variation along the  $z$  direction. This is a periodic 2D arrangement of skyrmion-like textures pointing up and down in a checkerboard pattern. The skyrmion density  $\Phi^z$  follows this pattern, in fact,  $\Phi^z = M^z/(4\pi|\mathbf{M}|^3)$  in this case. Thus for symmetry reasons, the total skyrmion number over the unit cell ( $\int dxdy\Phi^z$ ) vanishes, since positive and negative contributions cancel exactly. Now, a magnetic field is applied in the  $z$ -direction. We assume, in the interest of clarity, that this only adds a uniform magnetization  $(0, 0, m)$  to  $\mathbf{M}$ , but does not otherwise affect the spin arrangement. The skyrmion number per unit cell is an integer, and therefore can not change continuously with  $m$ . The behavior is shown in Fig. 1. The function is discontinuous at  $m = 0, \pm 2$  since it is at precisely these values of  $m$  that the spin texture develops nodes. These values can be easily obtained from the  $m = 0$  magnetization pattern of Eq. 4 by noting that the magnetization that is purely along the  $z$  axis takes on the values  $\mathbf{M} = 0, \pm 2\hat{z}$ . In the parameter regime  $0 < |m| < 2$ , a non-trivial, quantized spin chirality is obtained,  $\Phi^z = -\text{sgn}(m)$ .

At least in the limit of strong coupling between conduction electrons and magnetic background, it is natural that the spin direction  $\hat{\mathbf{n}}$  rather than the amplitude enters Eq. (3). However, in contrast to Refs. [1], we are concerned with a smoothly varying magnetization  $\mathbf{M}(\mathbf{r})$ , which has a spatially varying amplitude and even nodes. In the vicinity of nodes, electrons can not be infinitely sensitive to directional changes of the magnetic background and the theory needs to be extended. To include this effect in a simple way, let us assume that the electrons couple only then to the background spin texture, if the latter has an amplitude bigger than some cutoff. I.e. we consider  $\Phi_c = \Theta(\mathbf{M}^2 - M_0^2)\Phi$ ,

where  $\Theta$  is the Heaviside step function. As a result of this modification, the discontinuous jumps of the skyrmion number gets smoothed out, as shown in Fig. 1.

### 2.2. bcc1 spin crystal

We now consider the spin crystal bcc1. An explicit expression for  $\mathbf{M}(\mathbf{r})$  is given in Ref. [8]. Again, the average of the skyrmion density over a unit cell is zero for symmetry reasons [the point group symmetry of bcc1 is  $O(T)$  and therefore does not tolerate a nonzero axial vector like  $\langle\Phi\rangle$ ]. Hence, there are local skyrmion densities and conduction electrons will feel their effective fields, but the average effective field is zero. No anomalous Hall effect can therefore occur without an applied field.

Applying an external magnetic field lowers the symmetry and allows for  $\langle\Phi\rangle \neq 0$ . An external field has two effects on the bcc1 structure. First, it will induce a uniform magnetization  $\mathbf{m}$  pointing along the field and reduce the amplitude of the helical components. Second it will affect the relative amplitude and phases of the six interfering helical modes, thus transforming the spin texture. Eventually, the magnetic field will induce a transition to a different magnetic state, for example the conical single-spiral state or the spin-polarized (ferromagnetic) state. Here, we will only discuss the effect of a uniform component  $\mathbf{m}$  added to the otherwise unperturbed bcc1 structure.

Before we go into the details, we make a short remark about time reversal symmetry (T). The bcc1 state breaks T in such a way that it may not be restored by a subsequent translation [7,8,13]. As a consequence, there are two degenerate bcc1 states related by time reversal:  $\mathbf{M}(\mathbf{r}) \rightarrow -\mathbf{M}(\mathbf{r})$ . Let us label them by a parameter  $S = \pm 1$ . Time reversal symmetry requires that the integrated skyrmion density of the state  $S$  plus a uniform component  $\mathbf{m}$  satisfies

$$\langle\Phi\rangle_{S,\mathbf{m}} = -\langle\Phi\rangle_{-S,-\mathbf{m}} \quad (5)$$

In the following, we choose the lattice constant of the cubic (i.e. not elementary) unit cell as the unit of length and the amplitude of the elementary spiral modes which build up bcc1 as the unit of magnetization. The net effective field is determined by the integral of  $\Phi$  over the unit cell. It may be trivially written as

$$\langle\Phi^z\rangle = \int dz n_z(z) \quad (6)$$

where  $n_z = \int dxdy\Phi$  is an integer-valued function of  $z$ . It can only change its value at those positions of  $z$ , which correspond to nodes of  $\mathbf{M}$ . For  $\mathbf{m} = 0$ , the bcc1 structure has node lines, and  $n_z(z)$  could potentially change everywhere. Fortunately, we know from symmetry that in this case  $\langle\Phi\rangle = 0$ . For  $\mathbf{m} \neq 0$ , there are only discrete point nodes, and Eq. (6) turns out to be very useful to evaluate  $\langle\Phi^z\rangle$ . The other components,  $\langle\Phi^{x,y}\rangle$  may be obtained in a similar way.

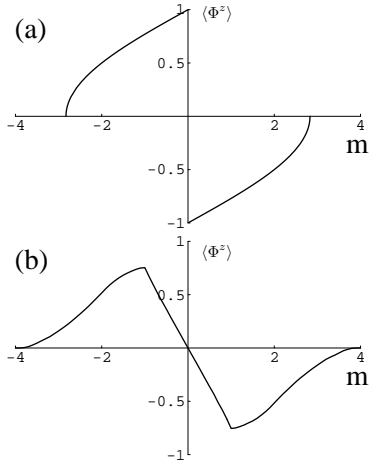


Fig. 2. Spin Chirality for bcc1 structure with a uniform magnetization  $(0, 0, m)$  superimposed (a) Shows  $\langle \Phi^z \rangle$  as function of  $m$ . (b) Same with a cutoff for  $M^2 < 1$ .

As an example, we consider the case  $\mathbf{m} = (0, 0, m)$ , i.e. a uniform component along  $\hat{\mathbf{z}}$  is added to the bcc1 texture (the case with the field along (111) direction is found to be qualitatively similar). For this geometry,  $\langle \Phi \rangle$  is an odd function of  $m$  and parallel to  $\hat{\mathbf{z}}$ , because of the  $O(T)$  point group symmetry. Hence, by Eq. (5), the result is independent of the time-reversal label  $S$ . The nodes of  $\mathbf{M}$  are particularly easy to analyze as a function of  $m$ . As a result,  $\langle \Phi^z \rangle$  may be calculated analytically. We find  $\langle \Phi^z \rangle = -\text{sgn}(m) (2/\pi) \arccos(|m|/2\sqrt{2})$  for  $0 < |m| < 2\sqrt{2}$  and  $\langle \Phi^z \rangle = 0$  elsewhere. The behavior is shown in Fig. 2. For comparison, the maximum spin density  $|\mathbf{M}|$  at  $m = 0$  is  $2\sqrt{3}$ . There are two phase transitions as a function of the uniform component. One is of course at  $m = 0$ , where  $\langle \Phi^z \rangle$  changes sign. However, the nontrivial transition occurs at the high-field ends, where  $\langle \Phi^z \rangle$  vanishes continuously but with an infinite slope. Note, at this point the magnetic structure is still non-trivial, but the 'spin-chirality' has been ironed out by the large uniform magnetization. Like in the former example, introducing a cutoff magnitude  $M_0$ , to model the decoupling of electrons from the low magnetization parts of the textures, smoothes these transitions to some degree. For the BCC1 spin crystal, with a spiral wavelength of  $170\text{\AA}$ , the 'spin-chirality' induced effective field is about 1.7 Tesla.

### 3. Summary

We have investigated the possibility of a topological Hall effect in non-trivial magnetic structures composed of skyrmion configurations under the influence of an external magnetic field. In two simple examples, we have analyzed the integrated skyrmion density  $\langle \Phi \rangle$ , which has been shown to act on conduction electrons as an effective internal field.

In both cases it is clear that an induced magnetization plays two opposite roles. On the one hand, it is required by symmetry to induce a non-zero spin chirality, but on the other hand, increasing it beyond a threshold value leads to a

vanishing  $\langle \Phi \rangle$ . This will ultimately lead to a non-monotonic effect of magnetization on the Hall conductivity, which is influenced by  $\langle \Phi \rangle$ . This, we believe is a hallmark of the spin chirality contribution to the Hall effect.

In the case of a two-dimensional spin structures, which may be realized for example in thin films, magnetic surfaces or interfaces, There is the possibility of a plateau region, where the effect is independent of the external field, due to quantization of the skyrmion number.

In both above cases, the effective field created by spin configurations turns out to be antiparallel to the uniform magnetization. This predicts the topological Hall effect to be of opposite sign to the normal Hall effect.

Questions left for future investigation include whether the 'topological' Hall effect discussed here is relevant to the experiments of [6], which are still on the low pressure side, and how electron transport is affected by a disordered, and possibly fluctuating, arrangement of non-trivial spin textures.

### 4. Acknowledgement

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